NATURAL CONVECTION FLOW IN A POROUS ENCLOSURE WITH LOCALIZED HEATING FROM BELOW

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\textbf{Abstract}

In this study steady natural convection flow in a two-dimensional fluid saturated porous enclosure with localized heating from below has been investigated. A portion of the bottom surface is heated and the symmetrically cooled the side walls and the top and rest of the bottom
walls are insulated. An implicit finite volume method with TDMA solver is used to solve the governing equations. Localized heating is simulated by a centrally located isothermal heat source on the bottom wall, and four different values of the dimensionless heat source length, 1/5, 2/5, 3/5 and 4/5 are considered. It is found that the flow field and the isotherm are symmetric owing to the symmetric boundary condition for the parameters considered here. It is also found that the increases of Rayleigh number and the heat source size enhance the heat transfer. The effect of heat source length and the Rayleigh number on streamlines and isotherms are presented, as well as the variation of the local rate of heat transfer in terms of the local Nusselt number from the heated wall. Finally, the average Nusselt number at the heated part of the bottom wall has been shown against Rayleigh number for the non-dimensional heat source length.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
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<tr>
<td>$L$</td>
<td>Enclosure height</td>
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<tr>
<td>$K$</td>
<td>Permeability of the porous medium</td>
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<tr>
<td>$Nu$</td>
<td>Local Nusselt number</td>
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<tr>
<td>$Nu_{av}$</td>
<td>Average Nusselt number</td>
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<tr>
<td>$Ra$</td>
<td>Darcy-Rayleigh number</td>
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<tr>
<td>$t$</td>
<td>Dimensional time</td>
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<tr>
<td>$T$</td>
<td>Fluid temperature</td>
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<tr>
<td>$T_C$</td>
<td>Temperature of the side walls</td>
</tr>
<tr>
<td>$T_H$</td>
<td>Temperature of the localised heat source</td>
</tr>
<tr>
<td>$x, y$</td>
<td>Dimensional coordinates</td>
</tr>
<tr>
<td>$X, Y$</td>
<td>Dimensionless coordinates</td>
</tr>
<tr>
<td>$u, v$</td>
<td>Dimensional velocity components along $x$ and $y$ directions</td>
</tr>
<tr>
<td>$U, V$</td>
<td>Dimensionless velocity components along $X$ and $Y$ directions</td>
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</table>
1. Introduction

The phenomenon of convective heat transfer in porous enclosures has attracted considerable interest of investigators as the transport process in a fluid where the motion is driven primarily by the interplay between density variation and the gravitational field is very common in several engineering and environmental problems. Applications of such analysis range from electronic equipment cooling, transpiration cooling, meteorology, geothermal exploitation, oil recovery, radioactive waste management in nuclear reactor systems, fire control and chemical, ground water pollution, storage of grain, fruits and vegetables, etc. Therefore natural convection appears naturally in several fields, where the heat to be dissipated is low enough and an attractive system in thermal control because of its low cost, reliability and simplicity of use.

The convective heat transfer in the rectangular/square enclosures has been studied extensively because of the above mentioned applications. A comprehensive review and extensive bibliography on natural convection in cavities can be found in Ostrach [1]. Most of the published studies on natural convection in rectangular cavities considered the classic problem of either vertically or horizontally imposed temperature gradient. Natural convection in a square cavity heated from below and cooled from one side has been studied by Anderson and Lauriat [2]. The authors observed only a single cell in the flow field and did not observe any Benard type instabilities. A further
study of the same configuration but a partially heated bottom wall with cooling from one side was studied by November and Nansteel [3]. It was reported in their studies that the fluid layer adjacent to the bottom wall remains attached up to the turning corner. The authors also claimed that convective heat transfer is shown to be most significant when slightly less than half of the lower surface is heated.

A numerical study of steady natural convection in rectangular cavity heated from below and symmetrically cooled from sides has been performed by Ganzarolli and Milanez [4]. In their numerical model the size of the cavity was varied from square to shallow. There are two heating boundary conditions have been employed on the bottom surfaces; isothermal and heat flux. The authors observed a distinct discrepancy between the isothermal and constant heat flux conditions for shallow cavity. They also observed that the cavity was not always thermally active along its whole length and the flow did not fill the cavity uniformly for isothermal heat source condition. However, the isotherms and streamlines occupy the whole cavity more uniformly, even for low values of the Rayleigh number for the constant heat flux condition.

Aydin and Yang [5] investigated the natural convection of a square cavity with localized heating from below with symmetrical cooling from the sidewalls. The top wall and the rest of the non heated portions of the bottom wall were considered adiabatic. The authors have shown the variation of the heat source length and the Rayleigh numbers. It is found in their simulation results that two counter rotating vortices were formed in the flow domain due to natural convection. The effect of the Rayleigh number and the length of the heat source on the average Nusselt number at the heated part of the bottom wall has been shown and discussed. Sharif and Mohammad [6] studied the same problem but with heat flux for heat source of the bottom surface instead of constant temperature.

On the other hand, the study of heat transfer in porous media has also got attention of many researchers. Neild and Bejan [7] and Ingham and Pop [8] contributed to an extensive overview of this important area of heat transfer in
porous media. There are many published investigations related to natural convection in rectangular porous enclosures [9-16] are available in the literature. Most of the work found in the literature deals with flow and heat transfer characteristics inside porous enclosures with constant wall temperatures. However, relatively little work has been done on the problem of natural convection in a fluid saturated porous enclosure with a discrete localised source of heat. Although recent works by Basak et al. [17] and Saeid [18] show the effect of a non-uniformly heated bottom wall inside a fluid-saturated porous enclosures. But they only considered continuous variation of the temperature along the wall. We take their work to the extreme by considering a discontinuous variation of the wall temperature profile.

In the present study, we are interested in investigating the unsteady natural convection laminar flow in a square cavity formed by insulated top and bottom walls, left and right walls being uniformly heated and cooled respectively. The basic equations of motion are transformed into a non-dimensional form, which are solved numerically by using an implicit finite volume method with TDMA solver is used to solve the governing equations. The effect of heat generation and the Rayleigh number on streamlines and isotherms are presented, as well as on the rate of heat transfer from the heated wall of the cavity are discussed and presented graphically. The Prandtl number has been chosen as unity in this study.

2. Formulation of Problem

Consider the flow of a Newtonian fluid within porous enclosure with height $L$ as shown in Figure 1. The non-dimensional governing equations are obtained with following assumptions: (a) The enclosure is completely filled with porous material, (b) Darcy’s law is assumed to hold, (c) the fluid is assumed to be a normal Boussinesq incompressible fluid, (d) negligible inertia effects, (e) the saturated porous medium is assumed to be isotropic in thermal conductivity, (f) the bottom wall has a centrally located heat source which is assumed to be isothermally heated at constant temperature $T_H$, the
side walls are isothermally cooled at a constant temperature $T_C$, while the bottom wall, except the heated part, and the top wall are considered to be insulated.

\[ \frac{\partial T}{\partial y} = 0 \]

Figure 1. Physical model and coordinate system.

Under the above assumption, the non-dimensional governing equations in terms of the stream function ($\psi$) and temperature ($\theta$) are as follows

\[ \frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -Ra \frac{\partial \theta}{\partial X}, \quad (1) \]

\[ \frac{\partial \theta}{\partial \tau} + \frac{\partial \psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}, \quad (2) \]

where the dimensionless variables are defined by

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \tau = \frac{t}{L^2 / \alpha}, \quad U = \frac{u}{\alpha / L}, \]

\[ V = \frac{v}{\alpha / L}, \quad \Psi = \frac{\psi}{\alpha}, \quad \theta = \frac{T - T_C}{T_H - T_C}, \quad Ra = \frac{g \beta \Delta T K}{\alpha L}. \quad (3) \]

The non-dimensional stream function, $\psi$, satisfies the following equations

\[ U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X}. \quad (4) \]
Equation (1) and (2) are subject to the following boundary conditions:

\[ \psi = 0 = 0 \text{ for } \tau = 0, \]
\[ \psi = 0, \theta = 0 \text{ for } 0 \leq Y \leq 1 \text{ at } X = 0, \]
\[ \psi = 0, \theta = 0 \text{ for } 0 \leq Y \leq 1 \text{ at } X = 1, \]
\[ \psi = 0, \theta = 1 \text{ for } \frac{1 - \varepsilon}{2} \leq X \leq \frac{1 + \varepsilon}{2} \text{ at } Y = 0, \]
\[ \psi = 0, \frac{\partial \theta}{\partial Y} = 0 \text{ for } 0 < X < \frac{1 - \varepsilon}{2} \text{ and } \frac{1 + \varepsilon}{2} < X < 1 \text{ at } Y = 0, \]
\[ \psi = 0, \frac{\partial \theta}{\partial Y} = 0 \text{ for } 0 \leq X \leq 1 \text{ at } Y = 1, \] (5)

where \( \varepsilon \) is the non-dimensional heat source length.

Once we know the numerical values of the temperature function we may obtain the rate of heat transfer in terms of the local Nusselt number, \( Nu \) from the heated portion of the bottom wall using the following relation:

\[ Nu = \left( \frac{\partial T}{\partial Y} \right)_{Y=0}. \] (6)

The average Nusselt number, \( Nu_{av} \) is given by

\[ Nu_{av} = \int_{\frac{1 - \varepsilon}{2}}^{\frac{1 + \varepsilon}{2}} \left( \frac{\partial T}{\partial Y} \right)_{Y=0} dX. \] (7)

The governing equations (1)-(2) along with the boundary conditions (5) are solved numerically, employing implicit finite volume method with TDMA solver. The Poisson like momentum equation (1) and the Energy equation (2) are discretised using the central difference but the time derivative is discretised using the three points backward difference formula to ensure the second order accuracy in both time and space, even though we have presented only steady state solution. After several grid independent test, 101 \( \times \) 101 non-uniform grids is used for the whole computation.
3. Results and Discussion

In the present study, the Prandtl number of the localized heating effect on the flow field and the isotherm has been considered in a square cavity with porous media. The working fluid has been chosen as $Pr = 1.0$. The Darcy Rayleigh number, $Ra$ is varied from $10^1$ to $10^3$ and the heat source size, $\varepsilon$ is varied from 0.2 to 0.8. Flow and temperature fields are presented in terms of streamline and isotherm contours respectively. Effects of the Rayleigh number and the heat source size on the heat transfer and fluid flow are observed. Latter, heat transfer performance is also examined in terms of average Nusselt number ($Nu$).

3.1. Benchmarking

Published experimental data are not available for the cavity configuration and boundary conditions similar to that undertaken in the present study. Thus, the validation of the computations against suitable experimental data could not be performed. However, in order to validate the predictive capability and accuracy of the present code, five published works have been chosen.

<table>
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<tr>
<th>Table 1. Comparisons of present numerical values</th>
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<tr>
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<tr>
<td>$Ra$</td>
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<tr>
<td>-----------------------------------------------</td>
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<tr>
<td>Baytas and Pop [9]</td>
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<tr>
<td>Moya et al. [10]</td>
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<tr>
<td>Walker and Homsy [16]</td>
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<tr>
<td>Present prediction</td>
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For validation purpose, a differentially heated square cavity has been considered. Average Nusselt number has been calculated and depicted in Table 1 for three different Darcy-Rayleigh number $Ra$ ($= 10^1, 10^2, 10^3$) and
compared with the earlier investigations [9-10, 14-16]. From Table 1, it is clearly revealed that the agreement between the present results and the previous results is very good indeed. So, it can be concluded that the present numerical method and the presented results are very accurate.

3.2. Effect of Rayleigh numbers

The evolution of the flow and the temperature fields in the enclosure for $Ra = 10^1, 10^2$ and $10^3$ are shown in Figure 2 for a representative case of $\varepsilon = 2/5$. Because of the symmetrical boundary conditions on the sidewalls, the flow and the temperature fields are symmetric about the vertical centerline of the enclosure. The flow rises in the center of the enclosure due to buoyancy effect and falls along each cold side walls creating mirror image structures that rotate clockwise in the right half and counterclockwise in the left half. It is noted that the horizontal component of velocity is zero everywhere along the midplane for these symmetric results. It can be seen that the shape of both cells are elliptical. Due to the symmetry, the flows in the left and the right halves of the enclosure are identical except for the sense of rotation. In each case, the flow rises along the vertical symmetry axis from the middle hot portion of the bottom wall and gets blocked at the top adiabatic wall, which turns the flow horizontally towards the isothermal cold walls. The flow then descends downwards along the cold sidewalls and turns back horizontally to the central region after hitting the bottom wall. Thus, the flowing fluid forms two symmetrical rolls with anticlockwise and clockwise rotations inside the enclosure. Since the Rayleigh numbers are small, viscous forces are more dominant than the buoyancy forces and hence, heat transfer is essentially diffusion dominated and the shape of the streamline tends to follow the geometry of the enclosure. The maximum values of the stream lines for $Ra = 10^1, 10^2, 5 \times 10^2$ and $10^3$ are 0.27, 3.64, 12.63 and 19.38, respectively. It is seen that as the Rayleigh number increases the flow becomes stronger. As a consequence, the stream lines for higher Rayleigh number ($Ra = 5 \times 10^2$ and $10^3$) are concentrated adjacent to the cold walls.
\[ Ra = 10^1, \psi_{\text{max}} = 0.266888, \psi_{\text{min}} = -0.266888 \]

\[ Ra = 10^2, \psi_{\text{max}} = 3.6376, \psi_{\text{min}} = -3.6376 \]

\[ Ra = 5 \times 10^2, \psi_{\text{max}} = 12.6298, \psi_{\text{min}} = -12.6298 \]
3.3. Effect of heat source size

The fluid flow and heat transfer behaviors with the change of discrete heat source size are investigated by performing numerical simulations for the square enclosure at different discrete heat source lengths of 0.2 to 0.8 are shown in Figure 3 for a representative case of $Ra = 10^3$. It is seen that the flow fields are qualitatively identical for different heat source size for a fixed Rayleigh number. However, quantitative results of the maximum values of the stream functions increase with increasing $\varepsilon$. The maximum values of stream function are 17.4405, 19.3829, 20.4939 and 21.0526 for $\varepsilon = 0.2$, 0.4, 0.6 and 0.8 respectively. One the other hand, the isotherms are affected by the increasing $\varepsilon$, as expected. Since the heated part of lower surface increases for the same Rayleigh number, the heating effect will be much more sensible. For a fixed $Ra$, with increasing $\varepsilon$, the flow field remains almost the same, while the temperature fields changes becoming more stratified for larger values of $Ra$.

Figure 2. Streamlines (left) and isotherms (right) for $\varepsilon = 2/5$. 
\( Ra = 10^3, \psi_{\text{max}} = 17.4405, \psi_{\text{min}} = -17.4405 \)

\( R a = 10^3, \psi_{\text{max}} = 19.3829, \psi_{\text{min}} = -19.3829 \)

\( R a = 10^3, \psi_{\text{max}} = 20.4939, \psi_{\text{min}} = -20.4939 \)
3.4. Local and average rate of heat transfer

The influence of the Rayleigh number on heat transfer for different heat source sizes has been plotted in Figure 4. Since the temperature field is symmetric, heat transfer is symmetrical with respect to symmetric centre line ($x = L/2$). It is inevitable that the higher Rayleigh number means more heat input. As a result, this heat intensifies the fluid convection. This means the fluid body receives more heat through convective heat transfer process. The most important thing is the heat transfer along the centre line. The heat transfer rate along this line is the minimum compared to both sides of the bottom wall. This has to happen as the symmetric behaviour of the temperature distribution along this line. As we can see in the stream function that there exist two symmetric cells on both sides of the centre line. Each cell acts like an insulator. It prohibits direct convective transfer between these two symmetric cells.
The average Nusselt number on the heated part of the bottom wall as a function of $Ra$ and $\varepsilon$ is presented in Figure 5. For a fixed $\varepsilon$, increases of Rayleigh number enhances heat transfer. This is obvious as the convection becomes important for higher $Ra$. For a fixed $Ra$, the Nusselt number also increases for increases of heat source size, $\varepsilon$ which is expected.
4. Conclusion

In the present study a numerical results of buoyancy induced flow and heat transfer in a two dimensional square cavity with localized bottom heating and symmetric cooled side walls has been investigated. Two main parameters, Rayleigh number ($Ra$) and local heat source size ($\varepsilon$) and their dependency on fluid flow and heat transfer have been analysed. It is revealed that the flow field and the isotherm are symmetric owing to the symmetric boundary condition for the parameters considered here. The increases of Rayleigh number and the heat source size enhance the heat transfer. For lower Rayleigh number the flow is dominated by conduction, however, as the Rayleigh number increases convection takes the role to dominant.

References


