Effects of corrugation frequency and aspect ratio on natural convection within an enclosure having sinusoidal corrugation over a heated top surface

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A B S T R A C T

Natural convection of a two-dimensional laminar steady-state incompressible fluid flow in a modified rectangular enclosure with sinusoidal corrugated top surface has been investigated numerically. The present study has been carried out for different corrugation frequencies on the top surface as well as aspect ratios of the enclosure in order to observe the change in hydrodynamic and thermal behavior with constant corrugation amplitude. A constant flux heat source is flush mounted on the top sinusoidal wall, modeling a wavy sheet shaded room exposed to sunlight. The flat bottom surface is considered as adiabatic, while the both vertical side walls are maintained at the constant ambient temperature. The fluid considered inside the enclosure is air having Prandtl number of 0.71. The numerical scheme is based on the finite element method adapted to triangular non-uniform mesh element by a non-linear parametric solution algorithm. The results in terms of isotherms, streamlines and average Nusselt numbers are obtained for the Rayleigh number ranging from \(10^3\) to \(10^6\) with constant physical properties for the fluid medium considered. It is found that the convective phenomena are greatly influenced by the presence of the corrugation and variation of aspect ratios.

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1. Introduction

Natural convection in enclosed space or closed cavities is under considerable attention of engineers and researchers because of its great importance in several thermal engineering problems. But enclosures with simple regular rectangular or triangular geometries have been studied in a great extend as such boundaries are easier to model and the hydrodynamic and thermal flow patterns and circulations are less complex than an enclosure having complex profile like wavy surface. Applications of such irregular enclosures can be found in designing heat exchangers, for example, double-wall thermal insulation to enhance the transfer of heat, solar collectors, electric machinery, cooling systems of micro-electronic devices, natural circulation in the atmosphere and so on.

Numerical simulation of natural convection is very complicated since the momentum and the thermal energy equations are coupled together due to the buoyancy force which causes the natural convective flow inside the domain. The investigation gets more challenging when it is for an enclosure. For an enclosure there are two regions. One is the boundary and the other is the core zone. The fact is that these two zones or regions have different thermal and flow characteristics for the same boundary conditions. This situation gets more complex if there are irregular or complex boundary profiles.

The review of the natural convection study was given by Ostrach [1]. The natural convection in enclosures having rectangular shapes has been studied in a great extend like Davis and Jones [2], Strada and Heinrich [3]. The natural convection heat transfer in a two-dimensional rectangular enclosure fitted with a periodic array of hot roughness element at the bottom was studied numerically by Amin [4] where the bottom surface was heated and the right vertical wall was cooled and the other walls were adiabatic. When the roughness element phase shift was equal to the half of its period, the heat transfer was found to be more significant for enclosures with higher values of roughness element amplitude. Besides, several earlier studies have also been performed for triangular enclosures by Akinsate and Coleman [5], Poulikakos and Bejan [6], Campo et al. [7], Varol et al. [8] and Holzmann et al. [9]. Saha and Khan [10] had a review study on the natural convection heat transfer in an attic-shaped space. Saha [11] concentrated on the study of unsteady natural convection in triangular cavity. Some more of similar trend of problems has been studied in Saha [12] and Saha et al. [13]. Asan and Namli [14,15] studied the buoyant flow in a triangular roof numerically for the resembling boundary conditions of winter and summer days. Others researchers have explored the analysis of natural convection in different complicated shapes of enclosures, such as trapezoidal enclosure by Moukalled.
and Acharya [16] and an enclosure having inclined roof by Das and Sahoo [17]. Morsi and Das [18] and recently Saha et al. [19] investigated a square enclosure with different shaped roof tops for constant surface temperature and constant heat flux heating, respectively. Saha et al. [20] performed numerical study of natural convection inside a modified square enclosure having elliptical top and bottom covers. Basak and Balakrishnan [21] studied the effects of thermal boundary conditions on natural convection flow within a square cavity.

Corrugation in the form of repeated non-linear profile is very important for heat transfer analysis as it has a significant thermal characteristic. There are different types of corrugations like sinusoidal, vee, trapezoidal etc. Each has different significant heat transfer enhancement ability. So convection in the presence of corrugation is of a great importance in thermal engineering. Noorshahi et al. [22] studied an enclosure with corrugated bottom surface maintaining at uniform heat flux and flat isothermal cooled top surface and side walls adiabatic. Their results showed that the pseudo-conduction increased with the wave amplitude increment. Natural convection along a vertical wavy surface was theoretically studied by Yao [23]. The observation showed that the local heat transfer rate as well as the average Nusselt number was smaller than that of the flat plate case and decreased with increase of the wave amplitude. Adjlout et al. [24] carried out an investigation on a numerical study of the effect of a hot wavy wall for differentially heated square cavity for different inclination angles, amplitudes and Rayleigh numbers. The average Nusselt number was lower than the square cavity. Mahmud et al. [25] showed the effect of surface waviness on natural convection heat transfer and fluid flow inside a vertical wavy walled enclosure for a range of wave ratio and aspect ratio (twice of the amplitude by wavelength). They observed that aspect ratio is the most important parameter for heat and fluid flows and higher heat transfer is obtained at a lower aspect ratio for a certain value of Grashof number. Das and Mahmud [26] analyzed the free convection inside both the bottom and the ceiling wavy and the isothermal enclosure. They indicated that, only at the lower Grashof number, the heat transfer rate rises when the amplitude wave length ratio changes from zero to other values. Mahmud and Islam [27] solved the laminar free convection and entropy generation inside an inclined wavy enclosure using SIP (Strongly Implicit Procedure) solver on a non-staggered grid arrangement. They obtained that the inclination angle of cavity affects the entropy generation due to the heat transfer and fluid friction. Dalal and Das [28] made a numerical solution to investigate the inclined right wall wavy enclosure with spatially variable temperature boundary conditions. Oztop [29] applied the elliptic grid generation to obtained sinusoidal duct geometry to enhance the forced convection heat transfer. Saidi et al. [30] presented numerical and experimental results of flow over, and heat transfer from, a sinusoidal cavity. They reported that the total heat exchange between the wavy wall of the cavity and the flowing fluid was reduced by the presence of vortex. The vortex plays the role of a thermal screen, which creates a large region of uniform temperature in the bottom of the cavity. Wang and Vanka [31] presented heat transfer and flow characteristics inside a wavy walled channel. Nishimura et al. [32] investigated flow characteristics such as flow pattern, pressure drop, and wall shear stress in a channel with symmetry, sinusoidal wavy wall. Asako and Faghri [33] gave a finite-volume prediction of heat transfer and fluid flow characteristics inside a wavy walled duct and tube, respectively. Mebrourk [34] studied the effect of wall waviness on heat transfer by natural convection in a horizontal wavy enclosure. Saha et al. [35] and Ganzarolli and Milanez [36] also studied the effects of different aspect ratios for a rectangular cavity. Whereas, in the former one the aspect ratio was changed for different enclosure height keeping the heated wall dimension constant; in the latter one, the aspect ratio was changed by changing the heated bottom wall. Varol and Oztop [37,38] studied the thermal and flow characteristics inside of a tilted solar collector having absorber at the wavy bottom surface for the aspect ratio of enclosure height and amplitude height.

From the above literature survey, it is found that few studies have been carried out for corrugated walled enclosure of different complexity. The main interest of this investigation is to study the heat transfer characteristics due to natural convection inside a summer day house exposed to sun light on the roof of that house. Thus a numerical model representing a modified enclosure with different sinusoidal corrugation frequencies of the wavy top surface and various aspect ratios of the enclosure by considering the constant heat flux at the top wall will be simulated numerically. The side walls are kept at a surrounding ambient temperature and the bottom wall is at adiabatic condition. There are two main objectives of the present study. First aim is to observe the comparative heat transfer characteristics of a modified square enclosure with wavy top surface of different corrugation frequencies. Effect of aspect ratio of the modified enclosure on the flow and heat transfer characteristics has also been investigated here as a part of second objective.

2. Mathematical model

Natural convection is governed by the differential equations expressing conservation of mass, momentum and energy. The
details of the geometry for the configuration considered here are shown in Fig. 1. The present model is a modified rectangular enclosure having sinusoidal corrugated top surface of 10% corrugation amplitude (\(a = 0.1\,L\)), maintained at uniform heat flux, \(q\). The surface profile of the sinusoidal top surface can be obtained by the following relation:

\[
y = H + a \sin\left(\frac{2\pi nx}{L}\right),
\]

where \(n\) is the corrugation frequency and the corresponding sinusoidal wavelength is \(\lambda = L/n\).

In the present study, a steady two-dimensional laminar flow of a viscous incompressible fluid is considered and the viscous dissipation term in the energy equation is neglected. Coupling between momentum and energy equations is made through the Boussinesq approximation which mainly relates the effect of density change due to change of temperature. Then the governing equations for steady natural convection can be expressed in the dimensionless form as follows:

\[
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0,  
\]

\[
U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{\partial P}{\partial x} + \Pr \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right),  
\]

\[
U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = -\frac{\partial P}{\partial y} + \Pr \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) + RaPr \Theta,  
\]

\[
\frac{\partial \Theta}{\partial x} + \frac{\partial \Theta}{\partial y} = \left(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2}\right),  
\]

where \(U\) and \(V\) are the non-dimensional velocity components in the \(X\) and \(Y\) directions, respectively, \(\Theta\) is the non-dimensional temperature, \(P\) is the non-dimensional pressure and \(Ra\) and \(Pr\) are the Rayleigh number and the Prandtl number, respectively. The Rayleigh number, Prandtl number and the aspect ratio are respectively defined as:

\[
Ra = \frac{g|\beta|L^4}{\nu^2 \alpha}, \quad Pr = \frac{\nu}{\alpha}, \quad \text{and } A_r = \frac{H}{L}.  
\]

The dimensionless parameters in the equations above are defined as follows:

\[
X \rightarrow \frac{x}{L}, \quad Y \rightarrow \frac{y}{L}, \quad U = \frac{uL}{\nu}, \quad V = \frac{vL}{\nu}, \quad P = \frac{\rho L^2}{\mu^2}, \quad \Theta = \frac{T - T_c}{\Delta T}, \quad \Delta T = \frac{qL}{\kappa}.  
\]

where \(\rho, \beta, \nu, \alpha, g\) and \(k\) are the fluid density, coefficient of volumetric expansion, kinematic viscosity, thermal diffusivity, gravitational acceleration and thermal conductivity, respectively. The corresponding boundary conditions for the above problem are given by:

All walls: \(U = V = 0\),

Bottom wall: \(\frac{\partial \Theta}{\partial Y} = 0\),

Right and left side walls: \(\Theta = 0\),

Top wall: \(\frac{\partial \Theta}{\partial Y} = -1\).

The average Nusselt number, \(Nu\) at the heated surface is a measure of convective heat transfer coefficient. The expression for the average Nusselt number is

\[
Nu = \frac{1}{\theta_c(X)} \int \frac{1}{\theta(X)} dX,  
\]

where \(\theta_c(X)\) is the local dimensionless temperature of the heated top surface. Another parameter of interest is the average fluid temperature inside the cavity which can be obtained as follows:

\[
\theta_{av} = \frac{1}{A} \int_{\lambda} \theta dA,  
\]

where \(A\) is the domain area of the 2D enclosure.

3. Finite element formulation

In the present study, the finite element formulation of the governing Eqs. (3)–(5) are similar to that used by Basak et al. [21]. Here a penalty parameter \(\gamma\) eliminates the pressure term \(P\) by using the continuity Eq. (2) as a constraint and hence the pressure distribution can be written as:

\[
P = -\gamma \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right).  
\]

Eq. (11) also satisfies the continuity Eq. (2) for large values of \(\gamma\). Now enforcing the above equation, the momentum equations, Eqs. (3) and (4) after elimination of \(P\) becomes

\[
U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \gamma \frac{\partial}{\partial x} \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) + Pr \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right),  
\]

\[
U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = \gamma \frac{\partial}{\partial y} \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) + Pr \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) + RaPr \Theta.  
\]
The basis set function \([N_k]\), is used to expand the velocity components \((U, V)\) and temperature \((\Theta)\) as

\[
U \approx \sum_{k=1}^{N} U_k N_k, \quad V \approx \sum_{k=1}^{N} V_k N_k, \quad \Theta \approx \sum_{k=1}^{N} \Theta_k N_k.
\]  

(14)

The following non-linear residual equations, Eqs. (15), (16), and (17) are then developed, by the Galerkin finite element method, respectively at nodes of internal domain \(A\):

\[
R_{fi}^{(1)} = \sum_{k=1}^{N} U_k \int_{A} \left( \left( \sum_{k=1}^{N} U_k N_k \frac{\partial N_k}{\partial x} \right) \frac{\partial N_k}{\partial x} + \left( \sum_{k=1}^{N} V_k N_k \frac{\partial N_k}{\partial y} \right) \frac{\partial N_k}{\partial y} \right) dX dY
\]

\[
+ \gamma \sum_{k=1}^{N} U_k \int_{A} \left( \frac{\partial N_k}{\partial x} \frac{\partial N_k}{\partial x} \right) dX dY + \sum_{k=1}^{N} V_k \int_{A} \left( \frac{\partial N_k}{\partial y} \frac{\partial N_k}{\partial y} \right) dX dY
\]

\[
+ \rho \delta \sum_{k=1}^{N} U_k \left( \frac{\partial N_k}{\partial x} \frac{\partial N_k}{\partial x} + \frac{\partial N_k}{\partial y} \frac{\partial N_k}{\partial y} \right) dX dY,
\]  

(15)

\[
R_{fi}^{(2)} = \sum_{k=1}^{N} V_k \int_{A} \left( \left( \sum_{k=1}^{N} U_k N_k \frac{\partial N_k}{\partial x} \right) \frac{\partial N_k}{\partial x} + \left( \sum_{k=1}^{N} V_k N_k \frac{\partial N_k}{\partial y} \right) \frac{\partial N_k}{\partial y} \right) dX dY
\]

\[
+ \gamma \sum_{k=1}^{N} U_k \int_{A} \left( \frac{\partial N_k}{\partial x} \frac{\partial N_k}{\partial x} \right) dX dY + \sum_{k=1}^{N} V_k \int_{A} \left( \frac{\partial N_k}{\partial y} \frac{\partial N_k}{\partial y} \right) dX dY
\]

\[
+ \rho \delta \sum_{k=1}^{N} V_k \left( \frac{\partial N_k}{\partial x} \frac{\partial N_k}{\partial x} + \frac{\partial N_k}{\partial y} \frac{\partial N_k}{\partial y} \right) dX dY
\]

\[
-(\text{RaPr}) \int_{A} \sum_{k=1}^{N} \Theta_k N_k dX dY,
\]  

(16)

\[
R_{fi}^{(3)} = \sum_{k=1}^{N} \Theta_k \int_{A} \left( \left( \sum_{k=1}^{N} U_k N_k \frac{\partial N_k}{\partial x} \right) \frac{\partial N_k}{\partial x} + \left( \sum_{k=1}^{N} V_k N_k \frac{\partial N_k}{\partial y} \right) \frac{\partial N_k}{\partial y} \right) dX dY
\]

\[
+ \sum_{k=1}^{N} \Theta_k \left( \frac{\partial N_k}{\partial x} \frac{\partial N_k}{\partial x} + \frac{\partial N_k}{\partial y} \frac{\partial N_k}{\partial y} \right) dX dY
\]  

(17)

Three points Gaussian quadrature formula is used for the evaluation of the integrals in the residual equations and using Newton’s method, the non-linear residual equations, Eqs. (15)–(17) are solved to determine the coefficients of the expansions in Eq. (14). The following relationships are then introduced:

\[
X = \sum_{k=1}^{6} X_k N_k(\xi, \eta) \quad \text{and} \quad Y = \sum_{k=1}^{6} Y_k N_k(\xi, \eta).
\]

(18)

where \(N_k(\xi, \eta)\) are the local six nodded triangular basis functions on the \(\xi - \eta\) domain. The integrals in the nonlinear residual equations, Eqs. (15)–(17) can be evaluated in \(\xi - \eta\) domain employing the following relationships:

\[
\frac{\partial N_k}{\partial \xi} = \frac{1}{J} \begin{bmatrix} \frac{\partial Y}{\partial \xi} & \frac{-\partial X}{\partial \xi} \\ \frac{\partial X}{\partial \xi} & \frac{\partial Y}{\partial \xi} \end{bmatrix} \frac{\partial N_k}{\partial \xi} \quad \text{and} \quad \frac{\partial N_k}{\partial \eta} = \frac{1}{J} \begin{bmatrix} \frac{\partial Y}{\partial \eta} & \frac{-\partial X}{\partial \eta} \\ \frac{\partial X}{\partial \eta} & \frac{\partial Y}{\partial \eta} \end{bmatrix} \frac{\partial N_k}{\partial \eta},
\]  

(19)

where, Jacobian matrix is

\[
J = \begin{vmatrix} \frac{\partial X}{\partial \xi} & \frac{\partial X}{\partial \eta} \\ \frac{\partial Y}{\partial \xi} & \frac{\partial Y}{\partial \eta} \end{vmatrix}.
\]  

(20)

The convergence criteria of successive solutions is fixed such that the relative error for each variable between consecutive iterations is calculated below the predefined value \(\delta\) as

\[
\sqrt{\frac{\sum_{m=1}^{\infty} \left( \frac{r_m - r_{m-1}}{r_{m-1}} \right)^2}{\sum_{m=1}^{\infty} \left( \frac{r_m}{r_{m-1}} \right)^2}} < \delta,
\]

(21)

where, \(m\) is the Newton’s iteration index and \(r = U, V, P, \text{and } \Theta\). The value of \(\delta\) is set to be \(10^{-4}\).

4. Grid refinements and validations

Non-uniform grids of six nodded triangular elements are employed here with denser grids clustering in regions near the heat source and the enclosed walls. To test and assess grid independence of the present solution scheme, many numerical runs are performed for higher Rayleigh number in order to determine optimum grid resolution of each set of simulation. Fig. 2 shows the grid refinement study performed for the case of \(n = 5\) and \(A_r = 1.0\) when \(Ra = 10^6\). From this experiment, it has revealed that a non-uniform spaced grid of 6927 elements is adequate to describe correctly the flow and heat transfer process inside the enclosure. In order to validate the numerical model, the results obtained from the similar model considering isothermal heating condition reported by Morsi and Das [18] are compared and presented in Table 1. The agreement is found to be excellent which indirectly validates the present computations and lend us confidence for the use of the present mathematical model.

<table>
<thead>
<tr>
<th>Rayleigh Number</th>
<th>Average Nusselt Number</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present</td>
<td>Morsi and Das [18]</td>
<td></td>
</tr>
<tr>
<td>(6 \times 10^3)</td>
<td>2.258</td>
<td>0.49</td>
</tr>
<tr>
<td>(8 \times 10^4)</td>
<td>2.456</td>
<td>0.37</td>
</tr>
<tr>
<td>(1 \times 10^4)</td>
<td>2.617</td>
<td>0.62</td>
</tr>
<tr>
<td>(2 \times 10^4)</td>
<td>3.171</td>
<td>0.63</td>
</tr>
<tr>
<td>(4 \times 10^4)</td>
<td>3.840</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Fig. 2. Grid refinement check using convergence of average Nusselt number with mesh elements for \(Ra = 10^6\), \(A_r = 1.0\) and \(n = 5\).
5. Results and discussion

The results of this study are obtained for the range of Rayleigh number is from $10^3$ to $10^6$ with the parametric variation of sinusoidal corrugation frequency, $n$ from 3 to 5 and the range of aspect ratio is $0.5 \leq Ar \leq 2.0$. Other governing parameter such as Prandtl number is fixed at 0.71 and non-dimensional corrugation amplitude ($a/L$) is set at 0.1. The resulting flow and thermal structures are analyzed to provide an idea or understanding underlying the mechanism behind the effect of the Rayleigh number, corrugation frequency and aspect ratio on the flow and the thermal fields and the heat transfer enhancement. Important dimensionless parameters of interest for the present study are the average Nusselt number, average fluid temperature and maximum cavity temperature which have been examined to make important conclusion of the present investigation.

5.1. Effect of corrugation frequency

In the following section, the influence of corrugation frequency of the top wavy wall of the modified enclosure with fixed aspect ratio ($Ar = 1.0$) on the flow and the thermal behaviour are thoroughly investigated.

5.1.1. Flow and thermal fields

The thermal fields and development of flow within the considered modified square enclosure having sinusoidal top surface for $Ra = 10^3$ and $10^6$ is presented in Figs. 3 and 4, respectively for different corrugation frequencies of $n = 3$, 4 and 5. The wavy top surface is heated by applying a constant heat flux there and has higher temperature than the surrounding walls. So the fluid near the top surface is less dense compared to the fluid adjacent to the adiabatic bottom wall as well as the vertical cold walls; as they are maintained at the constant ambient temperature resulting in a more or less symmetric thermal layers (see Fig. 3) and a two opposite circulation of the fluid inside the cavity as seen from Fig. 4.

Fig. 3 shows a thermal stratification behavior within the enclosure for all Rayleigh numbers. These thermal strata are oriented in such a manner that they seem to originate from the top section of the enclosure and then expanding in a hemispherical pattern. This hemispherical pattern for development of thermal layers is more apparent for lower Rayleigh numbers i.e. the layers for lower Rayleigh numbers are semi-circle or elliptical. Because for lower Rayleigh number, the heat transfer is purely by means of diffusion and the buoyancy force generated is not strong enough to initiate fluid convection. The aforementioned hemispherical pattern for the development of thermal strata tends to distort as the Rayleigh number increases i.e. the semi-circle or elliptical type thermal stratification completely distorts while $Ra = 10^6$. For higher Rayleigh number, the isotherm patterns maintain almost horizontal linear shape at the mid region of the enclosure. Since with the increase of Rayleigh number, the profiles of isotherms begin to change from ‘U’ shape to be more likely
inverted ‘T’ shape i.e. the isotherm contours follow horizontal straight line shape at the core region. Moreover, the buoyancy effect is becoming more apparent and a strong vertically downward force causes the thermal strata to change into a more flat shape at the mid section. This also proves the dominance of the convective currents due to the increase of Rayleigh number and hence the uniformity in temperature gradient decreases. Besides, the isotherm lines are very much packed and highly dense at the junction of the heated and the cold walls. This is evident from the large temperature gradient near the junction of the heated and the cold isothermal walls. The change of corrugation frequency does not provide any significant visual alteration of isotherm patterns. But as we know, with the increase of corrugation frequency, the heated surface area also increases and maximum temperature rises at the top of the cavity. This nature can be clearly identified from the value and the position of iso-temperature lines for all cases of corrugation frequency.

The fluid flow rises through the mid region of the enclosure and makes a clockwise rotation at the right segment and a counter clockwise rotation at the left segment of the cavity. For lower values of Rayleigh numbers such as of $10^3$, the convection effect is very less and hence the opposite flow is quite laminar as the buoyancy force is less, having two equal and opposite circulations on both sides of the cavity. Since the top surface is heated and two vertical side walls are cooled with adiabatic bottom surface, the flow should be symmetric along the geometric centerline ($X=0.5$). However, we can’t see the symmetric behavior. This is due to the fact that the top wavy surface is not symmetric along $X=0.5$. When the Rayleigh number increases, the buoyancy force increases. It causes a strong circulation of the fluid inside the cavity and the flow patterns get distorted due to this strong circulation. The convection heat transfer has been accelerated by the increment of buoyancy force and thus indicating convective heat transport as observed in Fig. 4. The streamline plots also show that the streamlines at the middle and near the side walls of the enclosure are more packed for lower Rayleigh numbers whereas the dense pack of streamlines can be seen at the upper top corners of the enclosure when $Ra=10^6$. As the buoyancy vector acts opposite to the gravity vector, the increment of buoyancy force pushes the stream lines to move upward direction. When the corrugation frequency increases, small eddies at the folded or ridged top section start to appear. With increase of corrugation frequency, those small eddies seem to be getting stronger and locally enhance the fluid flow. For lower corrugation frequency and low Rayleigh number, there are no small eddies but they normally appear as Rayleigh number increases. It is interesting to observe that when both $Ra$ and $n$ increase, these localized small eddies or bubbles appears more widely in all over the wavy pockets. In fact, the presence of

Fig. 4. Streamline patterns for different corrugation frequencies at $Ra=10^3$ and $10^6$ ($A_i=1.0$).
corrugation and the variation of its frequency enhance the velocity of the fluid close to the top surface. It is also observed that the vertical velocity component is comparatively higher at high Rayleigh numbers as it helps the circulation to move against the gravity. Due to the presence of corrugation profile at the top of the enclosure, the flow patterns lose their usual circular profile and streamlines near the wavy surface are then compressed and separated periodically with respect to the corrugation frequency which allows it to circulate more close to the heated top surface and enhance the heat transfer.

5.1.2. Heat transfer

The values of average Nusselt number have been evaluated for Rayleigh numbers in the range of $10^3 \leq Ra \leq 10^6$ and different corrugation frequencies $3 \leq n \leq 5$. The obtained values of $Nu$ are then plotted in Fig. 5(a). It is found that in case of low corrugation frequency, heat transfer rate are minimum in the modified enclosure. The increment of corrugation frequency at the top wall allows the convective currents to redirect more closely over the heated wavy surface, which are in turns enhanced by the increment of heated surface area and the presence of local eddies (see Fig. 4). Hence, the average Nusselt number improves drastically in all cases of Rayleigh number. In general, the average Nusselt number increases with the increase of Rayleigh number. But the net rise in $Nu$ due to change of corrugation frequency is much higher than that due to change of $Ra$.

Still it has observed from Fig. 5(a) that the average Nusselt number at $Ra = 10^6$ is higher than that at $Ra = 10^3$.

The dimensionless average fluid temperature ($Θ_{av}$) and maximum cavity temperature ($Θ_{max}$) have shown in Fig. 5(b) and (c) respectively. These figures clearly describe the effect of Rayleigh number on heat transfer characteristics. With the increase of Rayleigh number, both $Θ_{av}$ and $Θ_{max}$ decrease sharply for $Ra > 10^4$. When the mode of heat transfer is dominated by conduction for $10^3 \leq Ra \leq 10^4$, the decrement of $Θ_{av}$ and $Θ_{max}$ with $Ra$ is very low for all corrugation frequencies. Now the corrugation frequency also affects the value of $Θ_{av}$ and $Θ_{max}$ significantly. It is expected that due to high corrugation frequency, the heated surface area becomes larger and hence the net heat flow inside the cavity increases. Fig. 5(b) and (c) demonstrate this phenomenon clearly with the increase of corrugation frequency since both the values of $Θ_{av}$ and $Θ_{max}$ are higher when $n$ becomes high.

5.2. Effect of aspect ratio

In the following section, the influence of aspect ratio of the modified enclosure with the constant corrugation frequency ($n = 3$) on the flow and the thermal behaviour are thoroughly investigated.

5.2.1. Flow and thermal fields

The effects of the aspect ratio are briefly revealed with the help of isotherm and streamline plots as shown in Figs. 6 and 7 respectively. While Rayleigh number is considered to be kept unchanged, for higher aspect ratio, the isotherms are much developed compared to the isotherms for lower aspect ratio. At low Rayleigh number like $Ra = 10^3$, for $Ar \geq 1$, isotherms are developed in a hemispherical or elliptical shapes as in Fig. 6(b) and (c). This is obvious because for higher aspect ratio, there is enough space in longitudinal direction for the development of thermal stratiﬁcation but for the lower aspect ratio the isotherms are not that much developed (see Fig. 6(a)).

From Fig. 6(a), at $Ra = 10^3$ and $Ar = 0.5$, the isotherms near the cold walls are oriented in such a manner that they seem to be packed in layer for isotherm values from 0.1 to 0.3. This is an indication of weak mode of convective heat transfer. As $Ar$ increase i.e. for $Ar \geq 1$, these isotherms of magnitude from 0.1 to 0.3 move closer to the wavy top wall. This observation refers to ratiocination that as $Ar$ increase a higher temperature gradient is prevailed between the hot wavy top wall and the colt flat side walls, which indicates a more vigorous heat transfer. The higher the aspect ratio, the least the magnitude of the isotherms near the cold side walls.

An interesting observation from Fig. 6(f) for higher aspect ratio and Rayleigh number is that the isotherms near the heated wavy top surface shows characteristics of being under compression. But the isotherms near the adiabatic bottom wall seem to have their hemispherical or elliptical shapes as they had it for lower Rayleigh number. More particularly, when the distance along the side walls from the wavy top surface exceeds unity, the undistorted hemispherical manner for isotherms is prevailed. This illustration leads to the supposition that the aforementioned vertically acting force i.e. the buoyancy force seems to have more effective action at the upper section of the enclosure.

The streamline plots from Fig. 7 show that there are two opposite vortices as expected for each case. This is because the two side walls are maintained at cold temperature and the convective currents are separated into two flows after rising through the mid section of the enclosure. For a certain Rayleigh number, at lower aspect ratio like $Ar = 0.5$, Fig. 7(a), the vortices seem to be congested as for lesser space. As aspect ratio increase, these congested vortices start to attain an oval shape which indicates that the flow is well developed as there is enough space for the flow development. For higher aspect ratio like $Ar = 2.0$, Fig. 7(c), these oval shaped vortices are polarized close to the
heated wavy top surface. As for a certain aspect ratio, the density of the streamlines gets denser when the Rayleigh number increases; Fig. 7(d), (e) and (f), and the polarized vortices are shifted near the meeting place of the heated wavy top walls and the colder side walls which is very similar as described at the previous section except that the shifting of the vortices seems to be more noticeable in this case. This is obvious as the increment of Rayleigh number is the evidence of the domination of convection phenomenon. For higher Rayleigh numbers, for any aspect ratios, Fig. 7(d), (e) and (f), small vortices at the folded or ridged region of the wavy top surface become visible. They seem to get weaker as the aspect ratio increases.

5.2.2. Heat transfer

Fig. 8 represents the variation of average Nusselt number, average fluid temperature and maximum fluid temperature for all aspect ratios with the change of Rayleigh numbers. It shows that the nature of the response to the increment of Rayleigh number is the same for all aspect ratios. However, it is observed that the heat transfer phenomenon is greatly influenced by the change of aspect ratio of the enclosure at the lower aspect ratio such as the increase in heat transfer from $A_r = 0.5$ to $1.0$ (see Fig. 8a) but for $1.0 < A_r < 2.0$, the increase in heat transfer, if present there, is insignificant. The reason behind this behavior is that at the higher aspect ratio the vortices are polarized towards the upper region of the enclosure and the larger space available at lower region and the additional height of the cold walls comes to no use for heat transfer purpose. The average Nusselt number, $Nu$ and $\Theta_{\text{max}}$ are calculated at heated wall and the figures (Figs. 6, 7 and 8) show that these two parameters become invariant with change of $A_r$ after 1.0. This can only mean that the system reaches to a stable condition in order to exchange heat with the surrounding walls when each of the cold vertical walls have a length of $H = L$. Therefore, for higher aspect ratio, $1.0 < A_r < 2.0$ i.e. $H \geq L$, the increased length of cold side walls barely put a significance on the preexisting stability. Hence, any noticeable change in the average Nusselt number, $Nu$ and $\Theta_{\text{max}}$ is absent there i.e. the average Nusselt number, for all Rayleigh number, is almost same for the aspect ratio, $A_r = 1.0$ and 2.0.

Fig. 6. Isotherm patterns for different aspect ratios at $Ra = 10^3$ and $10^6$ ($n = 3$).
The dimensionless average temperature, $\Theta_{av}$, and the dimensionless maximum temperature, $\Theta_{\text{max}}$, have been plotted in Fig. 8(b) and (c), respectively. It is seen that, for all aspect ratios, with increase in Rayleigh number, the values for $\Theta_{av}$ and $\Theta_{\text{max}}$ decrease. For lower Rayleigh numbers like $10^3 < Ra < 10^6$, the decrease in $\Theta_{av}$ and $\Theta_{\text{max}}$ values are merely mentionable compared to $Ra > 10^4$, for all aspect ratios. For higher Rayleigh number i.e. $Ra > 10^4$, there is a drastic fall in $\Theta_{av}$ and $\Theta_{\text{max}}$ values. As aspect ratio increases, the decline rate for $\Theta_{av}$ and $\Theta_{\text{max}}$ increases. This high declining trend for $Ra > 10^4$ is more noticeable for the plot for $\Theta_{\text{max}}$, Fig. 8(c). Unlike the effect of corrugation frequency, both $\Theta_{av}$ and $\Theta_{\text{max}}$ have lower values for higher aspect ratio. For the plot for $\Theta_{\text{max}}$, Fig. 8(c), curves for $Ar = 1.0$ and $Ar = 2.0$ can barely be separately observed and the reason behind this has already been described above. All the curves in Fig. 8(c) seem to meet at the same value for $Ra = 10^6$.

6. Conclusions

The effects of corrugation frequency and aspect ratio on natural convection inside a sinusoidal corrugated enclosure is investigated and analyzed numerically. The study has been carried out for a modified rectangular enclosure with sinusoidal corrugation at the top surface. The penalty finite element method helps to obtain converged solutions which are then expressed in terms of stream functions and isotherm contours for $Ra = 10^3$ to $10^6$ and $Pr = 0.71$. From the above discussion, following conclusions can be drawn:

i. The study encompasses a constant value of Prandtl number 0.71 and a range of Rayleigh number from $10^3$ to $10^6$, representing the domination of convection over conduction heat transfer as the value of Rayleigh number increases.

ii. The sinusoidal corrugation at the heated top surface plays a significant role on the convection heat transfer mechanism. The higher the corrugation frequency ($n$), the more enhancement of heat transfers from the heated wall.

iii. The aspect ratio also plays a significant role on the convection heat transfer phenomenon. The increase in heat transfer from aspect ratio, $A_r = 0.5$ to $A_r = 1.0$ is excellent compared to the increase in heat transfer from $A_r = 1.0$ to $A_r = 2.0$. The increase in heat transfer from $A_r = 1.0$ to $A_r = 2.0$ is barely noticeable although the average temperature inside the cavity falls significantly with the increase of aspect ratios.
iv. For the case of lower aspect ratios, the average Nusselt number slightly decreases for Rayleigh number range $10^3 \leq Ra < 10^4$ but increases sharply for Rayleigh number $Ra > 10^4$.

v. Both the average and the maximum temperature show a decreasing trend with increase in Rayleigh number for all cases of different corrugation frequencies and aspect ratios.

vi. The value of average as well as maximum temperature increases with the increase of corrugation frequency and the decrease of aspect ratio.

References


