Radiation effects on natural convection laminar flow from a horizontal circular cylinder

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\ABSTRACT

The effect of radiation on natural convection flow from an isothermal circular cylinder has been investigated numerically in this study. The governing boundary layer equations of motion are transformed into a non-dimensional form and the resulting nonlinear systems of partial differential equations are reduced to convenient boundary layer equations, which are then solved numerically by two distinct efficient methods namely: (i) implicit finite difference method or the Keller-Box Method (KBM) and (ii) Straight Forward Finite Difference Method (SFFD). Numerical results are presented by velocity and temperature distribution of the fluid as well as heat transfer characteristics, namely the shearing stress and the local heat transfer rate in terms of the local skin-friction coefficient and the local Nusselt number for a wide range of surface heating parameter and radiation-conduction parameter. Due to the effects of the radiation the skin-friction coefficients as well as the rate of heat transfer increased and consequently the momentum and thermal boundary layer thickness enhanced.

Keywords: Natural convection; Radiation-conduction interaction; Finite difference method; Horizontal circular cylinder

1. Introduction

A medium is said to be optically dense if the mean free path of a radiation photon beam which travels through the medium is very small compared with the characteristic dimension of the medium. For an optically dense medium the radiative heat fluxes can be approximated by the Rosseland diffusion approximation \cite{1}, which has been greatly used in many radiation related studies. In the present work, the effects of thermal radiation with the Rosseland diffusion approximation on a free convection boundary layer flow from an isothermal cylinder have been investigated theoretically and numerically. The thermal radiation effects on the free convection flow are important in many engineering applications, such as in advanced types of power plants for nuclear rockets, high-speed flights, re-entry vehicles and processes involving high temperatures, and very little is known about the effects of radiation on the boundary-layer flow of radiating fluid past a body \cite{2, Ch. 13}.

At a high temperature the presence of thermal radiation alters the distribution of temperature in the boundary layer, which in turn affects the heat transfer
at the wall. In such situation the simultaneous treatment of the convective and radiative heat transfer is necessary. Cess [3] studied the interaction of thermal radiation with free convection heat transfer along a vertical flat plate by considering absorbing, emitting and non-scattering gas. The singular perturbation technique was used to solve the set of non-linear partial differential equations.

An analytical attempt was made to understand the non-equilibrium interaction between the thermal radiation and the laminar free convection from a heated vertical plate immersed in a radiating gas by considering Prandtl number $Pr = 1.0$ [4]. Cheng and Ozisik [5] investigated the radiation with free convection from a vertical plate, considering an absorbing, emitting and isotropically scattering fluid. A viscous, radiating and non-similar boundary layer flow from a stagnation region and a flat plate has been investigated by Shwartz [6]. He studied the behaviour for an emitting and absorbing gas including the entire range of optical thickness, from thin to thick.

Hossain et al. [7] have analyzed the effect of radiation using the Rosseland diffusion approximation, which leads to non-similarity solution for the forced and free convection flow of an optically dense viscous incompressible fluid past a heated vertical plate with uniform free stream velocity and surface temperature. Using a group of transformations, the boundary layer equations governing the flow were reduced to local non-similarity equations validating both in the forced and free convection regimes.

Radiation effects on the natural convection flow about a truncated cone had been studied by Yih [8] following the Rosseland diffusion approximation. Molla et al. [9,10] have investigated the radiation effect on mixed convection along a wavy surface and frustum of cone using the Rosseland diffusion approximation where the computational fluid was optically dense. Molla et al. [9,10] have found that the effects of radiation the rate of heat transfer from the wavy surface enhanced and consequently the thermal boundary layer increased.

Natural convection flow of viscous incompressible fluid from a horizontal circular cylinder represents an important problem, which is related to numerous engineering applications such as to handle hot wire, steam pipe, etc. It appears that Merkin [11] was the first to present a complete solution of this problem, using the Blasius and Gortler series expansion method along with an integral method and a finite-difference scheme. The problem of free convection boundary layer flow on a cylinder of elliptic cross-section was also studied by Merkin [13]. Ingham [14] investigated the boundary layer flow on an isothermal horizontal cylinder. Nazar et al. [15] have investigated the natural convection flow along a uniformly heated horizontal circular cylinder considering the micro polar fluid. They described the effects of micro-rotation of the fluid on the heat transfer and the skin-friction coefficient. To the best of our knowledge, radiation effects on free convection flow from an isothermal horizontal circular cylinder have not yet been studied and the present work demonstrates this issue.

In the present study, it is proposed to investigate the natural convection flow of an optically dense viscous incompressible fluid past an isothermal horizontal circular cylinder, considering the Rosseland diffusion approximation. The basic equations of motion are transformed into convenient forms, which are solved numerically using a very efficient finite-difference scheme together with the Keller-box method [16] and the straight forward finite difference method. Consideration is given to the situation where the buoyancy forces assist the natural convection flow for various combinations of the radiation-conduction parameter $R_d$ and the surface heating parameter $\theta_s$. The numerical results allow us to predict the different behaviour that can be observed when the relevant parameters are varied.

2. Formulation of problem

A steady two-dimensional laminar free convective flow from an isothermal circular cylinder of radius $a$, which is immersed in a viscous and incompressible optically dense fluid, is considered. It is assumed that the surface temperature of the cylinder is $T_w$, where $T_w > T_\infty$. Here $T_\infty$ is the ambient temperature of the fluid and $T$ is the temperature of the fluid. The physical configuration considered is as shown in Fig. 1.

Under the usual Boussinesq approximation, following Merkin [11] the equations governing the flow are

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0,$$

$$\rho \left( \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} \right) = \mu \frac{\partial^2 \bar{u}}{\partial y^2} + \rho g \beta (T - T_\infty) \sin \left( \frac{\pi y}{a} \right),$$

$$\frac{\partial \bar{T}}{\partial x} + \frac{\partial \bar{T}}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 \bar{T}}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial \bar{q}_r}{\partial y},$$

where $(\bar{u}, \bar{v})$ are the velocity components along the $(x, y)$ axes, $g$ is the acceleration due to gravity, $\rho$ is the fluid density, $k$ is the thermal conductivity, $\beta$ is the coefficient of thermal expansion, $\mu$ is the viscosity of the fluid, $C_p$ is the specific heat at constant pressure,
where $\theta$ is the Stefan-Boltzmann constant, $y$ is the Grashof number, and $\delta$ is the non-dimensional temperature function. Substituting the variables (6) into equations (1)–(4) lead to the following non-dimensional equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(7)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{Pr} \frac{\partial^2 u}{\partial y^2} + 0 \sin x$$

(8)

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \left(1 + 3 \frac{R_d}{\delta} \left(1 + \Delta \theta^3 \right)^{\frac{3}{2}} \frac{\partial \theta}{\partial y} \right)$$

(9)

and the corresponding boundary conditions are

$$u = v = 0, \quad \theta = 1 \quad \text{at} \quad y = 0$$

(10a)

$$u \to 0, \quad \theta \to 0 \quad \text{as} \quad y \to \infty$$

(10b)

where $R_d$ is the radiation-conduction parameter or Planck number, $\theta_w$ is the surface heating parameter and $Pr$ is the Prandtl number, which are defined respectively as

$$R_d = \frac{4 \sigma T_{\infty}^3}{k(x_r + \sigma_s)}, \quad \theta_w = \frac{T_w}{T_{\infty}}, \quad \Delta = \theta_w - 1 \quad \text{and} \quad Pr = \frac{\mu C_p}{k}.$$

(11)

3. Numerical methods

Investigating the present problem the authors have employed two numerical methods, namely, implicit finite difference Method or Keller box method (KBM) are elaborately described by Cebeci and Bradshaw [17] and the straight forward finite difference (SFFD), which are individually described below.

3.1. Implicit Finite Difference or Keller Box Method (KBM)

To solve Eqs. (7)–(9), subject to the boundary conditions (10), we assume the following transformations

$$\psi = \psi(x, y), \quad \theta = \theta(x, y),$$

(12)

where $\psi$ is the non-dimensional stream function defined in the usual way as
\[
\frac{\partial u}{\partial y} = v = -\frac{\partial \psi}{\partial x}.
\] (13)

Substituting (13) into Eqs. (7)–(10) and after some algebraic manipulations, the transformed equations take the following form

\[
\frac{\partial^3 f}{\partial y^3} + f \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \sin x \frac{\partial f}{\partial \theta} = x \left(\frac{\partial f}{\partial \psi} \frac{\partial f}{\partial \psi y} - \frac{\partial f}{\partial \psi x} \right),
\] (14)

\[
\frac{1}{Pr} \frac{\partial}{\partial y} \left[ \left\{ 1 + \frac{4}{3} R_d (1 + (\theta_w - 1) \theta^2) \right\} \frac{\partial \theta}{\partial y} \right] + f \frac{\partial \theta}{\partial y} = x \left(\frac{\partial f}{\partial \psi y} \frac{\partial \theta}{\partial \psi x} - \frac{\partial f}{\partial \psi x} \frac{\partial \theta}{\partial \psi y} \right),
\] (15)

along with the boundary conditions

\[f = \frac{\partial f}{\partial y} = 0, \quad \theta = 1 \text{ at } y = 0,\] (16a)

\[\frac{\partial f}{\partial y} \rightarrow 0, \quad \theta \rightarrow 0 \text{ as } y \rightarrow \infty.\] (16b)

It can be seen that near the lower stagnation point of the cylinder, i.e. when \(x \approx 0\), Eqs. (14) and (15) reduce to the following ordinary differential equations

\[f''' + f'' - f^2 + \theta = 0,\] (17)

\[\frac{1}{Pr} \left[ \left\{ 1 + \frac{4}{3} R_d (1 + (\theta_w - 1) \theta^2) \right\} \theta' \right] + f \theta' = 0,\] (18)

subject to the boundary conditions,

\[f(0) = f'(0) = 0, \quad \theta(0) = 1,\] (19a)

\[f' \rightarrow 0, \quad \theta \rightarrow 0 \text{ as } y \rightarrow \infty.\] (19b)

In the above equations, primes denote differentiation with respect to \(y\).

The physical quantities of principle interest are the shearing stress and the rate of heat transfer in terms of the skin-friction coefficient \(C_f\) and the Nusselt number \(Nu\) respectively, which can be written as

\[C_f = \frac{(\tau_w)_{y=0}}{\mu U^2_x} \quad \text{and} \quad Nu = \frac{a \quad q_w + q_r}_{y=0} \frac{k(T_w - T_\infty)}{k(T_w - T_\infty)},\] (20)

where \(\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)\) and \(q_c = -k \left(\frac{\partial T}{\partial y}\right)\). (21)

Using the variables (5), (6), (13) and the boundary condition (16a) into Eqs. (20) and (21), we get

\[C_f Gr^{\frac{1}{4}} = x \frac{\partial^2 f(x, 0)}{\partial y^2},\] (22)

\[Nu Gr^{\frac{1}{4}} = - \left(1 + \frac{4}{3} R_d \theta^2 \right) \frac{\partial \theta(x, 0)}{\partial y}.\] (23)

The results of the velocity and temperature distributions are calculated respectively from the following relations

\[u = x \frac{\partial f}{\partial y}, \quad \theta = \theta(x, y).\] (24)

A very efficient and accurate implicit finite difference method (the Keller box method) is employed to solve the nonlinear system of partial differential equations (14) and (15). Eqs. (14) and (15) are written in terms of first order equations in \(y\), which are then expressed in finite difference form by approximating the functions and their derivatives in terms of the central differences in both coordinate directions. Denoting the mesh points in the \((x, y)\) plane by \(x_i\) and \(y_j\), where \(i = 1, 2, 3, \ldots, M\) and \(j = 1, 2, 3, \ldots, N\), central difference approximations are made such that the equations involving \(x\) explicitly are centred at \((x_{i-1/2}, y_{j-1/2})\) and the remainder at \((x_{i+1/2}, y_{j-1/2})\), where \(y_{j-1/2} = (y_j + y_{j-1})/2\), etc. This results in a set of nonlinear difference equations for the unknowns at \(x_i\) in terms of their values at \(x_{i-1}\). These equations are then linearised by the Newton’s quasi-linearization technique and are solved using a block-tridiagonal algorithm, taking as the initial iteration of the converged solution at \(x = x_{i-1}\). Now to initiate the process at \(x = 0\), we first provide guess profiles for all five variables (arising the reduction to the first order form) and use the Keller box method to solve the governing ordinary differential equations. Having obtained the lower stagnation point solution it is possible to march step by step along the boundary layer. For a given value of \(x\), the iterative procedure is stopped when the difference in computing the velocity and the temperature in the next iteration is less than \(10^{-5}\), i.e. when \(|\delta f| \leq 10^{-5}\), where the superscript denotes the iteration number.

3.2 Straight forward finite difference (SFFD)

The new transformations for the SFFD

\[X = x, \quad Y = y, \quad U = \frac{u}{x}, \quad V = v,\] (25)

Using (25) into (7)–(10), yields
\[
X \frac{\partial U}{\partial X} + U \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial Y} = 0,
\]

\[
XU \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + U^2 \frac{\partial^2 U}{\partial Y^2} + \frac{\theta \sin X}{X},
\]

\[
XU \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{\Pr} \frac{\partial}{\partial Y} \left[ \left\{ \left( 1 + \frac{4}{3} R_d (1 + \theta(w-1)0)^3 \right) \frac{\partial \theta}{\partial Y} \right\} \right].
\]

The corresponding boundary conditions are

\[
U = V = 0, \quad \theta = 1 \quad \text{at} \quad X = 0 \quad \text{any} \quad Y,
\]

\[
U = V = 0, \quad \theta = 1 \quad \text{at} \quad Y = 0, \quad X > 0,
\]

\[
U \to 0, \quad \theta \to 0 \quad \text{as} \quad Y \to \infty, \quad X > 0.
\]

Now Eqs. (26)–(28) subject to the boundary conditions (29) are discretised using the central-difference for diffusion terms and the forward-difference for the convection terms, finally we get a system of tri-diagonal algebraic equations. The algebraic equations have been solved by double sweep technique. The numerical discretisation of Eqs. (26)–(28) are given below:

\[
V_{ij} = V_{ij-1} - \Delta Y \frac{1}{2} (U_{ij-1} + U_{ij}) - X_i \frac{\Delta Y}{\Delta X} (U_{ij} - U_{ij-1}),
\]

\[
\text{for} \quad j = 2 \ldots N, \quad i = 1 \ldots M.
\]

The momentum equation

\[
X_i U_i \left( \frac{U_{ij} - U_{ij-1}}{\Delta X} \right) + V_i \left( \frac{U_{ij} - U_{ij-1}}{2 \Delta Y} \right) + U_{ij}^2 = \left( \frac{U_{ij-1} - 2U_{ij} + U_{ij+1}}{\Delta Y^2} \right) + \frac{\theta_{ij} \sin X_i}{X_i}.
\]

And the energy equation

\[
X_i U_i \frac{\theta_{ij} - \theta_{ij-1}}{\Delta X} + V_i \left( \frac{\theta_{ij+1} - \theta_{ij-1}}{2 \Delta Y} \right) = \frac{1}{\Pr} \frac{\partial}{\partial Y} \left[ \left\{ 1 + \frac{4}{3} R_d (1 + \theta(w-1)0)^3 \right\} \frac{\theta_{ij+1} - \theta_{ij-1}}{2 \Delta Y} \right]
\]

where \(N\) and \(M\) are the maximum numbers of \(Y\) and \(X\)-points respectively.

The computation is started from \(X = 0.0\), and then marches up to the upper stagnation point of the circular cylinder \((X \approx \pi)\). Here \(\Delta x = \pi/180\) and \(\Delta y = 0.01\) are used for the \(X\)-and \(Y\)-grids, respectively. Now it can be calculated the skin-friction coefficient and the rate of heat-transfer from the following dimensionless relations:

\[
C_d Gr^{1/4} = X \frac{\partial U}{\partial Y} (y=0),
\]

\[
NuGr^{-1/4} = - \left( 1 + \frac{4}{3} R_d \theta^3 \right) \frac{\partial \theta}{\partial Y} (y=0).
\]

### 4. Results and discussion

The numerical results for the skin-friction coefficient \(C_d Gr^{1/4}\) and the Nusselt number \(NuGr^{-1/4}\) are obtained for representative values of the radiation-conduction parameter \(R_d\) (= 0.0, 0.2, 0.5, 0.8, 1.0) and surface heating parameter \(\theta_w\) (=1.1, 1.3, 1.6, 1.9) against the curvature parameter \(x \in [0, \pi]\). It should be noted that for both CO\(_2\) in the temperature range of 37.77–343.33°C (with the corresponding Prandtl number range 0.76–0.6) and NH\(_3\) vapour in the temperature range of 48.88–204.44°C (with the corresponding Prandtl number 0.88–0.84) the value of \(R_d\) at 1 atm ranges from 0.033 to 0.1, whereas for water vapour in the temperature range of 104.44–482.22°C (with the corresponding Prandtl number \(Pr \approx 1.0\)), the \(R_d\) value lie between 0.02 and 0.3 (see [3]). It should be noted that without radiation effect \((R_d = 0.0)\), we recover the problem that discussed by Merkin [11] and Nazar et al. [15] considering \(Pr = 1.0\) which is shown in Table 1.

The numerical results of the skin-friction coefficient \(C_d Gr^{1/4}\) and the Nusselt number \(NuGr^{-1/4}\) for different values of the radiation-conduction parameter \(R_d\) (= 0.0, 1.0, 2.0, 3.0) while \(\theta_w = 1.1\) and \(Pr = 0.73\) are illustrated in Figs. 2(a) and (b), respectively. Here we notice that the agreement between the results obtained by using the KBM and the SFFD is excellent indeed. From the figures, it can be seen that an increase in radiation-conduction parameter \(R_d\) leads to an increase in the skin-friction coefficient \(C_d Gr^{1/4}\) and the Nusselt number \(NuGr^{-1/4}\). This may be attributed to the fact that the increase of the values of \(R_d\) implies more interaction of radiation with momentum and thermal boundary layers.

In Tables 2 and 3, the results of the skin-friction coefficient \(C_d Gr^{1/4}\) and the Nusselt number \(NuGr^{-1/4}\) are shown, respectively, for different values of surface heating parameter \(\theta_w\) (=1.1, 1.3, 1.6, 1.9), while \(R_d = 0.5\) and \(Pr = 0.73\). Again the comparisons between the two solutions of KBM and SFFD method are found to be in excellent agreement. We further notice that an increase
in the values of the surface heating parameter \( \theta_w \) leads to an enhancement in the results of \( C_f \), \( Gr^{1/4} \), and \( NuGr^{-1/4}/C_0^{1/4} \). For example, at \( x = \pi/2 \), the skin-friction coefficients \( C_f \), \( Gr^{1/4} \), and \( NuGr^{-1/4}/C_0^{1/4} \) increase by 11.28% and 59.87%, respectively, while \( \theta_w \) increases from 1.1 to 1.9. This phenomenon can easily be understood from the fact that when the surface heating parameter \( \theta_w \) increases, the temperature of the surface rises and the thickness of the thermal boundary layer grows. Therefore, the surface rate of heat transfer, that is the Nusselt number \( NuGr^{-1/4}/C_0^{1/4} \), increases. Again, this temperature increase of the fluid corresponds to the high

![Fig. 2](image_url)

(a) Skin-friction and (b) rate of heat transfer for different values of \( R_d \) while \( \theta_w = 1.1 \) and \( Pr = 0.73 \).

Table 2
The results of \( C_fGr^{1/4} \) for different values of surface heating parameter \( \theta_w \) while \( R_d = 0.5 \) and \( Pr = 0.73 \)

<table>
<thead>
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<th>( \theta_w )</th>
<th>( x )</th>
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<th>SFFD</th>
<th>KBM</th>
<th>SFFD</th>
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<th>SFFD</th>
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Table 3
The results of $\text{NuGr}^{-1/4}$ for different values of surface heating parameter $\theta_w$ while $R_d = 0.5$ and $Pr = 0.73$

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Fig. 3. (a) Velocity and (b) temperature distribution for different $R_d$ while $y_w = 1.1$ and $Pr = 0.73$ at $x = \pi/3$.

Fig. 4. Streamlines for (a) $R_d = 0.0$ (b) $R_d = 0.5$ (c) $R_d = 1.0$ while $\theta_w = 1.1$ and $Pr = 0.73$. 
surface shear stress which augments the skin-friction coefficient $C_f \delta_x^{1/4}$.

Attention is now given to the effects of pertinent parameters on the dimensionless velocity and temperature in the flow field, computed only by the KBM, and these are presented graphically in Fig. 3. Figs. 3(a) and (b) illustrate the velocity and temperature distributions against $y$ for different values of the radiation-conduction parameter $R_d (= 0.0, 0.2, 0.5, 0.8, 1.0)$ at $x = \pi/3$ while $Pr = 0.73$ and $\theta_w = 1.1$. These figures display how $R_d$ influences on the fluid velocity and temperature. As $R_d$ increases, the velocity and temperature gradients at the surface increase which again enhances the fluid velocity and temperature.

Figs. 4 and 5 illustrate the effect of the radiation-conduction parameter $R_d$ on the development of streamlines and isotherms, which are plotted for $Pr = 0.73$ and $\theta_w = 1.1$. From Fig. 4(a), it is seen that without the effect of radiation (i.e., $R_d = 0.0$) the non-dimensional value of $\psi_{max}$ within the computational domain is about 2.75 near the downstream point ($x \approx \pi$) of the cylinder and when the boundary layer thickness is the lowest, but $\psi_{max}$ increases with the increment of $R_d$ and it attains about 4.20 for $R_d = 1.0$ (see Fig. 4(c)). This phenomenon fully coincides with the early discussion made on Fig. 3(a), the fluid speeds up as $R_d$ increases and the thickness of the velocity boundary layer also increases. The isotherm patterns for corresponding values of $R_d$ are shown in Fig. 5. From these three frames, we can see that the growth of thermal boundary layer over the surface of the cylinder is significant. As $x$ increases from the lower stagnation point ($x \approx 0.0$), the hot fluid rises due to the gravity, hence the thickness of the thermal boundary layer, $y_t$ increases. This phenomenon is very straightforward as can be seen in this frame for $R_d = 1.0$ in 5(c). In this case the fluid temperature increases slightly which was also noticed in Fig. 3(b).

5. Conclusion

The effect of radiation on natural convection flow from an isothermal circular cylinder has been investigated numerically. The governing boundary layer equations of motion are transformed into a non-dimensional form and the resulting nonlinear systems of partial differential equations are reduced to convenient boundary layer equations, which are then solved numerically by two distinct efficient methods namely (i) Implicit Finite Difference Method or the Keller-box method and (ii) Straight Forward Finite Difference Method (SFFD). From the present investigation the following conclusions may be drawn:

- The skin-friction coefficient and the Nusselt number increase when the value of the radiation-conduction parameter $R_d$ increases.
- As $R_d$ increases, both the velocity and the temperature distribution increase significantly at $x = \pi/3$ of the surface.
- An increase in the values of $\theta_w$ leads to an increase in the values of the skin-friction coefficients and the Nusselt number.
- For increase values of $R_d$, the momentum and thermal boundary layer increase significantly.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>Radius of the circular cylinder</td>
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<tr>
<td>$C_p$</td>
<td>Specific heat at constant pressure</td>
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Greek symbols

\( \alpha_r \)  Rosseland mean absorption coefficient
\( \beta \)  Volumetric coefficient of thermal expansion
\( \psi \)  Stream function
\( \tau_w \)  Wall shearing stress
\( \rho \)  Fluid density
\( \mu \)  Dynamic viscosity of the fluid
\( \nu \)  Kinematic viscosity of the fluid
\( \theta \)  Dimensionless temperature function
\( \theta_w \)  Surface heating parameter

References